

I.D. #
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Subject Physics 560
Course Section
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Date

Receiving or giving aid in a final examination is a cause for dismissal from the University.

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## Lecture 15:

### 1.) Review

The Stark ladder shows that an  $\mathcal{E}$ -field is not sufficient to yield transport in a crystal. Some mechanism must be explicitly included to relax the momentum.

$$\dot{p} = -eE - \frac{m\dot{v}}{\tau} \quad \text{where} \quad \boxed{\dot{v} = \frac{\partial \mathcal{E}}{\partial \hbar k}}$$

You will solve this as a HW problem.

### 2.) Transport: Phonon Scattering

It is an expt. fact that at low  $T$ , the  $\sigma$ -conductivity scales as

$$\rho \sim \begin{cases} T^5, & T < T_D \\ T, & T > T_D. \end{cases}$$

I want to give you a flavor of how to derive this result. The full derivation is in 11.5.1 - 11.5.3 in my book. The key quantity we will compute is the scattering rate. To introduce this quantity, let's look at the Boltzmann eq which is based on the distribution function:  $f(k, r, t)$ .  $f(k, r, t)$  is the probability that a quantum state is occupied with a momentum  $k$ , position,  $r$  and a time  $t$ . This type of description is valid at long wavelengths  $\lambda \gg \hbar v_F / k_B T$ , otherwise the uncertainty principle is violated. Consider the volume element

$$d\Omega = \frac{d^3 r d^3 k}{(2\pi\hbar)^3}$$

$f d\Omega = \#$  of electrons in  $d\Omega$ .

$\frac{df}{dt} = 0$  if no collisions take place.

$\frac{df}{dt} = \left. \frac{\partial f}{\partial t} \right|_{\text{collisions}}$  = Change with respect to time of the distribution when collisions obtain.  $f$  is an explicit fcn. of  $k, r, t$ .

$$\Rightarrow \frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{k} \cdot \nabla_k f + \dot{r} \cdot \nabla_r f$$

There should be no dependence on time ( $\frac{\partial f}{\partial t} = 0$ ) because all volumes are equivalent. For a homogeneous system  $\nabla_r f = 0$ .

$$\Rightarrow \left(\frac{\partial f}{\partial t}\right)_{\text{coll.}} = \dot{k} \cdot \nabla_k f$$

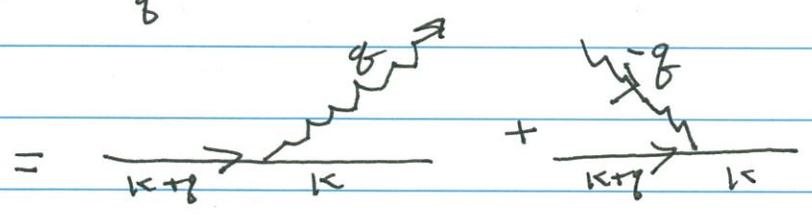
Semiclassically  $\dot{k} = -eE$ . When phonons are providing the scattering mechanism, we need to include them in the distribution function. Call this  $g(r, k, t)$ . For phonons  $\nabla_k g = 0$  because there are no forces on phonons. Also  $\frac{\partial g}{\partial t} = 0$ . The combined eqs. are

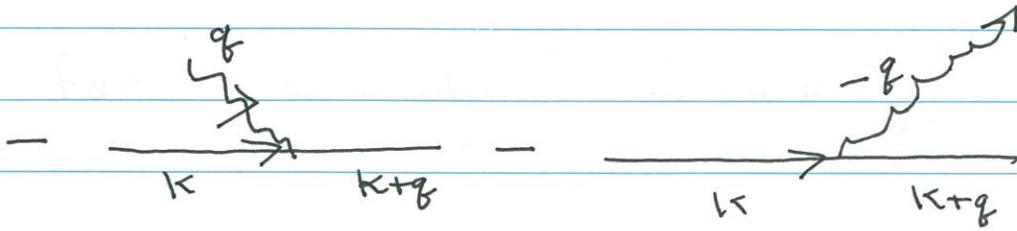
$$\left(\frac{\partial g}{\partial t}\right)_{\text{coll.}} = \dot{r} \cdot \nabla_r g$$

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll.}} = \dot{k} \cdot \nabla_k f$$

To help solve the equation for  $f$ , we need to set up a gain-loss equation

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll.}} = \sum_g (\text{gain} - \text{loss})_{\text{in state}}$$





All of this is done explicitly in 11.5.1 - 11.5.3. There are a number of approximations

### a.) Relaxation-time approximation

Let  $f_k = n_k + \delta f_k$   
F.D. dist.

ansatz:  $\left(\frac{\partial f}{\partial t}\right)_{\text{coll.}} = -\frac{\delta f_k}{\tau}$

recall  $\left(\frac{\partial f}{\partial t}\right)_{\text{coll.}} = k \cdot \nabla_k f$

### b.) Linearization

$$\nabla_k f = \nabla_k (n_k + \delta f_k)$$

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drop this term

$$= \nabla_k n_k$$

$$= \frac{\partial n_k}{\partial E_k} \nabla_k E_k$$

$$= -\frac{\partial n_k}{\partial \mu} \cdot \nabla_k E_k$$

for  $\epsilon_k = \frac{(\hbar k)^2}{2m}$ ,  $\nabla_k \epsilon_k = \frac{\hbar \vec{k}}{m}$ .

$$= - \frac{\delta f}{\tau} = -e E \cdot \frac{\hbar \vec{k}}{m} \frac{\partial n_k}{\partial \epsilon}$$

$$V = \hbar/m.$$

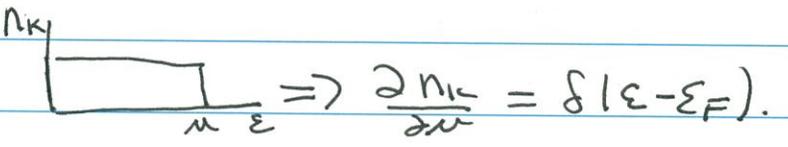
$$\Rightarrow \delta f = -e \vec{E} \cdot \vec{V} \frac{\partial n_k}{\partial \epsilon} \tau.$$

Define the current as

$$\vec{J} = -e \int d^d k \vec{V}_k f_k$$

$\Rightarrow$  The conductivity is defined as

$$\sigma_{\alpha\beta} = \frac{\partial j_\alpha}{\partial E_\beta} = e^2 \int \frac{d^d k}{(2\pi\hbar)^d} \tau |\hbar k| V_\alpha V_\beta \frac{\partial n_k}{\partial \epsilon}$$

at  $T=0$   $n_k$  is   $\Rightarrow \frac{\partial n_k}{\partial \epsilon} = \delta(\epsilon - \epsilon_F).$

recall,  $d^d k = d\Sigma d k_\perp$

$$= d\Sigma \frac{d\epsilon_k}{\frac{d\Sigma_\perp}{dk_\perp}}$$

$$= d\Sigma \frac{d\epsilon_k}{\nabla_k \epsilon_k}$$

So all we need is  $\zeta(\omega)$ . The final expression is

$$\frac{1}{\zeta} = T^5 \int_0^{T_D/T} x^5 \frac{\partial}{\partial x} \frac{dx}{(e^x - 1)}$$

$T \ll T_D$

$T \gg T_D$

$$T^5 \int_0^\infty x^5 \frac{\partial}{\partial x} \frac{1}{(e^x - 1)} dx$$

$$= T^5 5! \zeta(5)$$

$$T^5 \int_0^{T_D/T} x^5 \frac{\partial}{\partial x} \left[ \frac{1}{x} + \dots \right] dx$$

$$T^5 \int_0^{T_D/T} x^3$$

$$= \frac{1}{4} \left( \frac{T_D}{T} \right)^4 T^5$$

$$= \boxed{\# T}$$

linear in  $T$ !

There are 3 distinct contributions to  $T^5$ .

- a.) Phonons from  $\omega \rightarrow T^3$
- b.) momentum transfer,  $T$
- c.) fraction of electrons involved  $T/T_D$ .